

# Sidewall Muffler Design for Pulsed Exciplex Lasers

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Sidewall mufflers are used in pulsed exciplex lasers to rapidly attenuate pressure waves and restore cavity medium homogeneity at successive pulses. The technical basis for designing these is reviewed with primary focus on the near-cavity region and emphasis on physical understanding. The approach combines rudimentary theory and flow visualization, starting with simple situations and building toward a more complete picture. Important results are as follows: longitudinal wave attenuation in a finite capacity muffler involves a balance between precursor pulse decay and recompression wave strength which defines an optimum configuration; acoustic absorber is required in the backing volume, and a basis for choosing its resistivity is indicated; the contribution of a close-in muffler to transverse wave decay within the laser cavity is also characterized. Further, an entropy wave is shown to be generated within a muffler which is confined to the vicinity of the sidewalls. Other entropy sources in a closed loop flow system are noted.

## I. Introduction

OVER the past few years there has been rapid development of lasers using mixtures of rare gases and halides<sup>1</sup> as well as metal vapors and halides.<sup>2</sup> These are known collectively as exciplex lasers and they operate at visible and near-ultraviolet wavelengths. Such lasers are typically pulsed ( $t_p \sim 1 \mu s$ ), involve repetitively pulsed operation to achieve high average output power, and require flow through the cavity to remove waste-heated gas and maintain lasing medium optical homogeneity at successive pulses.<sup>3,4</sup> An interesting set of conditions<sup>5</sup> involve Mach 0.2–0.3 flow, an initial overpressure  $\Delta p_c/p \sim 0.3$  immediately after rapid waste energy deposition, medium clearing within 10–20 acoustic transits across the cavity, and baseline homogeneity  $(\delta\rho/\rho)_{rms} \sim 1 \times 10^{-5}$ . Closed-cycle flow systems are of predominant interest due primarily to the expense of the laser gases involved.

Rapid acoustic attenuation is a key ingredient for achieving high rep-rate in such devices and this must be accomplished without adverse impact on baseline flow homogeneity. Sidewall mufflers are generally accepted as the best approach, though the arrangement of these, relative to the cavity, has been a subject of some controversy.<sup>5,6,7</sup> A close-in muffler configuration is shown schematically in Fig. 1. This typically involves the most demanding acoustic design considerations for good performance. There are also issues in integrating it with other components near the cavity, e.g., an e-beam for pumping the laser gas, magnets to control the e-beam and hence improve laser efficiency, and optics for light extraction. On the other hand, a close-in arrangement offers potential for reducing attenuator and flow loop size and hence system cost. Canted cavity sidewalls are also shown in Fig. 1. These are required to speed clearing of transverse acoustic waves from the cavity, at least for the opposed e-beam configurations that are typical of efficient XeF\* and KrF\* lasers.

This paper is meant to provide an overview of the design considerations, evolved over the past several years, for sidewall mufflers in pulsed exciplex lasers. No attempt will be made to catalog a definitive design process, which seems premature since the modeling is not validated to that level and the data base is limited. Space limitations have dictated that many technical details be omitted in order to allow a

reasonably complete, though hardly exhaustive survey. In particular, the primary focus is on the near cavity region. Reference 5 provides more in-depth coverage and background for the interested reader, and a recent summary article is available from Cassady.<sup>8</sup> Also, the topics and emphasis herein reflect the author's own background and experience.

Longitudinal pressure wave attenuation is the most meaningful starting point for the discussion. Such waves arise when the initial cavity overpressure is relieved by a pressure pulse moving into the upstream and downstream muffler sections, in the form of a shock followed by a rarefaction from the two edges of the energy deposition region. This topic is addressed the simplest sense in Sec. II using quasi-one-dimensional theory. More detailed understanding requires description of two-dimensional phenomena, both in terms of achieving performance as projected in Sec. II and of attenuating transverse acoustic waves due to nonuniform energy deposition in the laser cavity. Preliminary results in this area are covered in Sec. III. The final topic, entropy waves arising from pressure wave interaction with flow and attenuator elements, is discussed in Sec. IV. The main focus is on entropy disturbances arising within the upstream muffler and their confinement to the vicinity of the sidewalls. This is followed by conclusions in Sec. V.

## II. Guidelines from Quasi-One-Dimensional Theory

### Background

A key point concerning sidewall mufflers is that their length scales with the local hydraulic diameter of the flow channel,  $D$ . This was first evident in a correlation of experimental data for an infinite capacity muffler.<sup>9</sup> In that case the shock attenuation distance, and hence the muffler length, increases linearly with  $D$ . Sidewall reactance, grazing flow, and two-dimensional effects alter the form of this dependence,<sup>5</sup> particularly during the later stages of shock attenuation. However, these do not alter the conclusion that there can be an advantage to placing the mufflers close to the laser cavity in the flow loop section of minimum hydraulic diameter. Another thing found for an infinite capacity muffler is that the shock attenuation distance varies inversely with the open area ratio  $\alpha$ . There is a counter-balancing consideration in the case of finite capacity mufflers, required for closed cycle lasers, as will be seen.

Pulsed excimer lasers involve much weaker shocks than those considered in Ref. 9, with shock Mach number  $M_s \leq 1.1$  being typical. It is then quite plausible to consider weak wave

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approximations. This involves assuming small perturbations about the baseline flow state, that shocks propagate at the baseline sound speed, and that the mass exchange process at the perforated sidewalls can be considered locally incompressible. A simpler theory results, but the main benefit is the existence of a similarity parameter in the weak wave regime. As seen in Fig. 2, the Wu and Ostrowski data<sup>10</sup> for three different initial shock Mach numbers can be correlated by a single line, as predicted earlier,<sup>11</sup> when plotted in terms of weak wave variables. Note that the discharge coefficient is lower than normal due, it is believed, to grazing flow and sidewall reactance effects.<sup>5</sup> A reduction in the number of parameters in weak wave theory allows an overview of the design strategy to be more easily displayed.

#### Isolated Muffler Optimization

A theoretical tradeoff study has been performed for an isolated muffler configuration bounded upstream and downstream by semi-infinite, constant area, hard-walled ducts. That is, the reflection of waves by each muffler across the laser cavity to its neighbor, etc., will be disregarded (cf., Fig. 1.) This is predicated on a relatively small reflection coefficient and is acceptable as a first approximation. In the process, parameters specifying the geometric location of the mufflers relative to the cavity, as well as performance characteristics of the other muffler, are eliminated. Weak wave theory is used for the same reason and the closely spaced baffle approximation is made; i.e., the muffler is treated as locally reacting.<sup>12</sup> Resistance of acoustic absorber in the backing volume is also not considered. That would only influence the mass exchange process for a locally reacting muffler.

The basic purpose behind these idealizations is making the tradeoff study more tractable while retaining the most essential physics controlling typical sidewall muffler designs. In other words, the objective is preliminary design definition. Parameters entering the study are given in Figure 3. The backing volume is finite and the configuration is two-dimensional. Also, the assumed rectangular input waveform is representative of the initial pressure pulse produced in an excimer laser with wavelength  $\lambda \sim \ell_c$ , the cavity length in the flow direction.

Results of the tradeoff study are given in Fig. 4 for a baseline flow Mach number of 0.2. The characteristic attenuation distance in the weak wave regime is

$$L = (H/2\alpha C_d) \sqrt{\Delta p_i / 2\gamma p} \quad (1)$$

Note that  $\lambda/L$  is proportional to the open area ratio  $\alpha$ . Thus, the solid curves in Fig. 5 can be viewed as displaying transmitted wave amplitude vs  $\alpha$  for various values of the backing parameter  $2h/H$  and the relative muffler length  $\ell_m/\lambda$ . In each plot the leftmost curve (dashed) is that associated with precursor shock decay, plotted in different form in Fig. 2, and can be considered to correspond to  $2h/H = \infty$ .

For a small open area ratio, the solid curves follow the shock decay curve, but the transmitted wave amplitude begins

increasing as  $\alpha$  gets larger. There is a tendency to believe that this is due to finite backing volume effects on shock decay. However, that is not true because the shock decay curve is followed when the input waveform is triangular. Instead, with a rectangular waveform, the peak transmitted wave strength is associated with the trailing rarefaction and that is affected by the backing depth. Leading shock decay depends only on mass exchange processes occurring in the vicinity of the perforated sidewall for a locally reacting muffler.

An explanation of transmitted wave strength variation seen in Fig. 4 is as follows. During the time the pressure in the flow channel is high (as imposed by the input signal), the backing volume fills and the pressure therein increases. When the pressure in the flow channel drops as the input signal passes by, the flow direction through the perforated walls reverses and the gas in the flow channel is recompressed to a level commensurate with the maximum overpressure in the backing volume. It is thus intuitively plausible that the recompression wave strength decreases with  $2h/H$  for fixed  $\alpha$  and increases with  $\alpha$  for fixed  $2h/H$ , ultimately becoming larger than the attenuated incident wave for sufficiently large  $\alpha$  or  $\lambda/L$ . Both trends are seen in Fig. 4. Evidently, there is an optimum configuration (i.e.,  $\alpha$ ) for a finite capacity muffler which balances precursor pulse decay vs recompression wave strength.

The results in Fig. 4 for optimum muffler configurations can be correlated approximately as a power law relationship for the transmission factor

$$\frac{\Delta p_t}{\Delta p_i} \sim \left( \frac{H}{2h} \frac{\lambda}{\ell_m} \right)^{0.7} \quad (2)$$

with a coefficient of one. Note that  $H\lambda/2h\ell_m$  can be interpreted as the ratio of cavity volume to the backing volume of the muffler. Equation (2) is the first of the scaling relations and provides the basis for muffler sizing.

Studies at a range of Mach numbers, including those in Fig. 4, have established that the normalized wave residence time  $at_m/L \sim 3.2-3.3$  for an optimum configuration with  $2h/H \geq 1$ , where  $t_m$  is the time to transit the muffler,  $a$  is the baseline

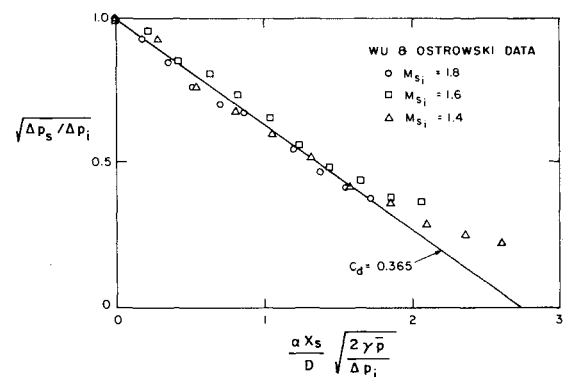


Fig. 2 Shock decay in terms of weak wave variables.

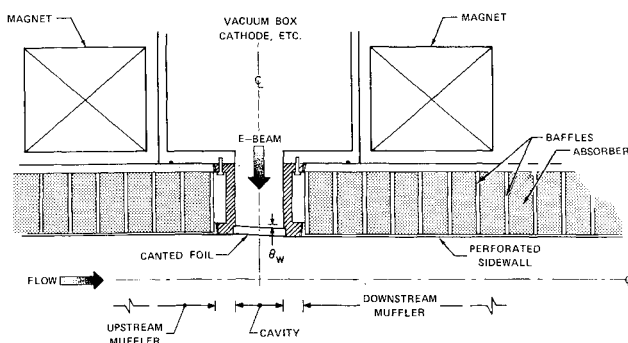


Fig. 1 Close-in muffler configuration.

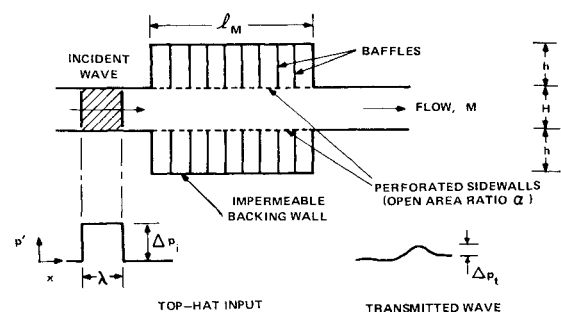


Fig. 3 Parameters entering muffler tradeoff study.

sound speed, and where  $L$  is defined in Eq. (1). This is slightly less than the value  $\alpha t_m/L=4$  required for complete shock decay according to weak wave theory.<sup>11</sup> Thus, relating muffler length to residence time for an upstream running wave,

$$(\ell_m/L)/(1+M) \sim 3.2-3.3 \quad \text{at optimum} \quad (3)$$

The appropriate open area ratio can be estimated accordingly with  $M>0$  downstream, where allowance of negative  $M$  is merely a convenient artifice. One interpretation of the Mach number dependence is that the mass exchange time must be adjusted to reflect true residence time of the pulse in the muffler section.

It appears likely that the optimum  $\alpha$  would alter if based on a round trip through the muffler rather than a single pass. However, the change would be small for low Mach number operation. Also, the strong exterior reflection required to make a round-trip assessment meaningful must generally be avoided in actual laser systems. Exterior reflectors can contribute to entropy waves and standing acoustic waves, the former influencing mostly beam degradation and the latter primarily beam jitter. The recommended preliminary design basis is as given above.

Mufflers with spacially varying open area ratio and backing depth were also considered as part of the isolated muffler optimization study. The conclusions were that the results derived for constant  $\alpha$  and  $2h/H$  apply more generally provided it is understood that Eq. (3) determines the average open area ratio over the muffler length and that the quantity entering the denominator in Eq. (2) is the actual muffler backing volume.

#### Global Conservation

Global conservation relations provide another perspective on what can and cannot be done with sidewall mufflers. Simple forms can be derived from the weak wave equations only for zero baseline flow,<sup>5</sup> but these are approximately valid more generally. Constant flow channel area will also be assumed and a locally reacting muffler.<sup>12</sup> The latter essentially eliminates the significance of resistive material in the backing volume on global conservation. Locally reacting designs are of primary interest for a close-in muffler configuration.

For an isolated muffler in which quiescent conditions prevail before arrival of a  $P$ -wave input of infinite duration, the relations are

$$\begin{aligned} \int_0^\infty P'_i dt &= \int_0^\infty P'_t dt, & \int_0^\infty Q'_r dt &= 0 \\ \int_0^\infty (P'_t)^2 dt + \int_0^\infty (Q'_r)^2 dt & \\ + \frac{4(\Delta p_i)^2}{L} \int_0^\infty \int_0^{\ell_m} \left| \frac{p' - p'_b}{\Delta p_i} \right|^{3/2} dx dt &= \int_0^\infty (P'_t)^2 dt \end{aligned} \quad (4)$$

where  $P' = p' + \rho a u'$  and  $Q' = p' - \rho a u'$  are proportional to the linearized Riemann variables, subscripts  $i$ ,  $t$ , and  $r$  denote the incident, transmitted, and reflected waves, respectively, and  $\Delta p_i$  is a characteristic amplitude. These relations, as derived, are restricted to constant flow channel height, but the open area ratio and backing depth can vary with  $x$ . The backing depth does not appear explicitly. Its influence is present nonetheless because it dictates the magnitude of  $p' - p'_b$ .

Note that the impulse of the transmitted wave is always equal to that of the incident wave according to first of Eqs. (4), predicated on the assumptions which underlie it. That is

$$\int_0^\infty P'_t dt = \ell_c \Delta p_c / a$$

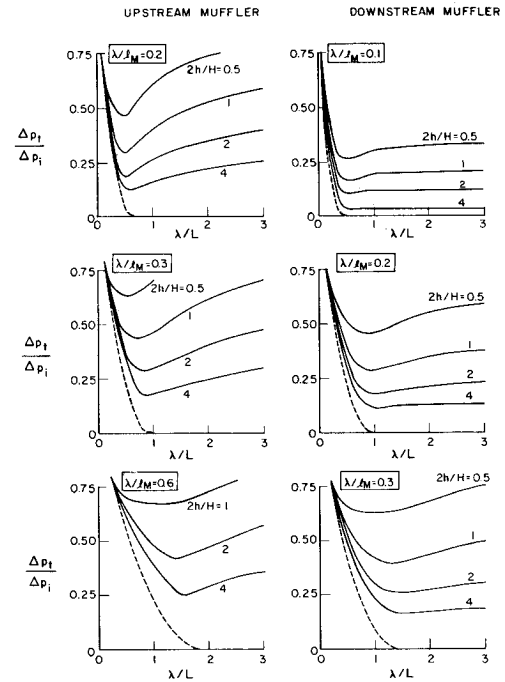


Fig. 4 Tradeoff study results for transmission factor at  $M = \pm 0.2$ .

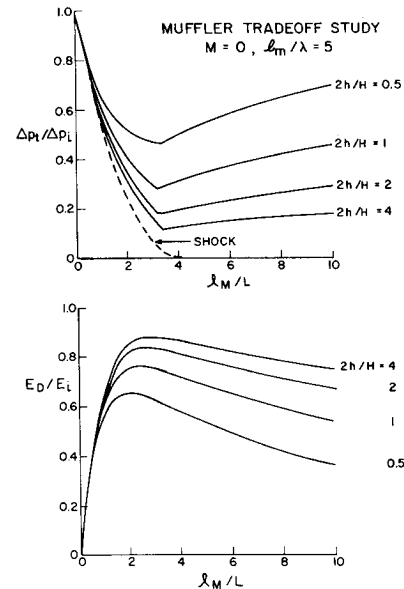


Fig. 5 Sidewall muffler performance in terms of transmission factor and fraction of acoustic energy dissipated.

for a cavity length  $\ell_c$  in the flow direction and initial overpressure  $\Delta p_c = 2\Delta p_i$  immediately after energy loading. It is thus never possible to prevent waves getting out of the sidewall muffler sections into the rest of a low- $\Delta p$  closed loop; that would require a bulk absorber in the flow stream. To reduce the amplitude of transmitted waves to an acceptable level, their wavelength must be stretched to a large dimension compared to the cavity so that the impulse can be preserved. This generally requires  $\ell_m$  substantially larger than the input wavelength,  $\lambda \sim \ell_c$ . Another interpretation of the counterbalancing trends for the transmitted precursor pulse and recompression wave, seen in Fig. 4, is now evident: there are limits to wavelength stretching for given muffler dimensions.

The total impulse of the reflected wave, going back into the laser cavity, is always zero under the assumptions made in deriving Eq. (4). That immediately raises the possibility of

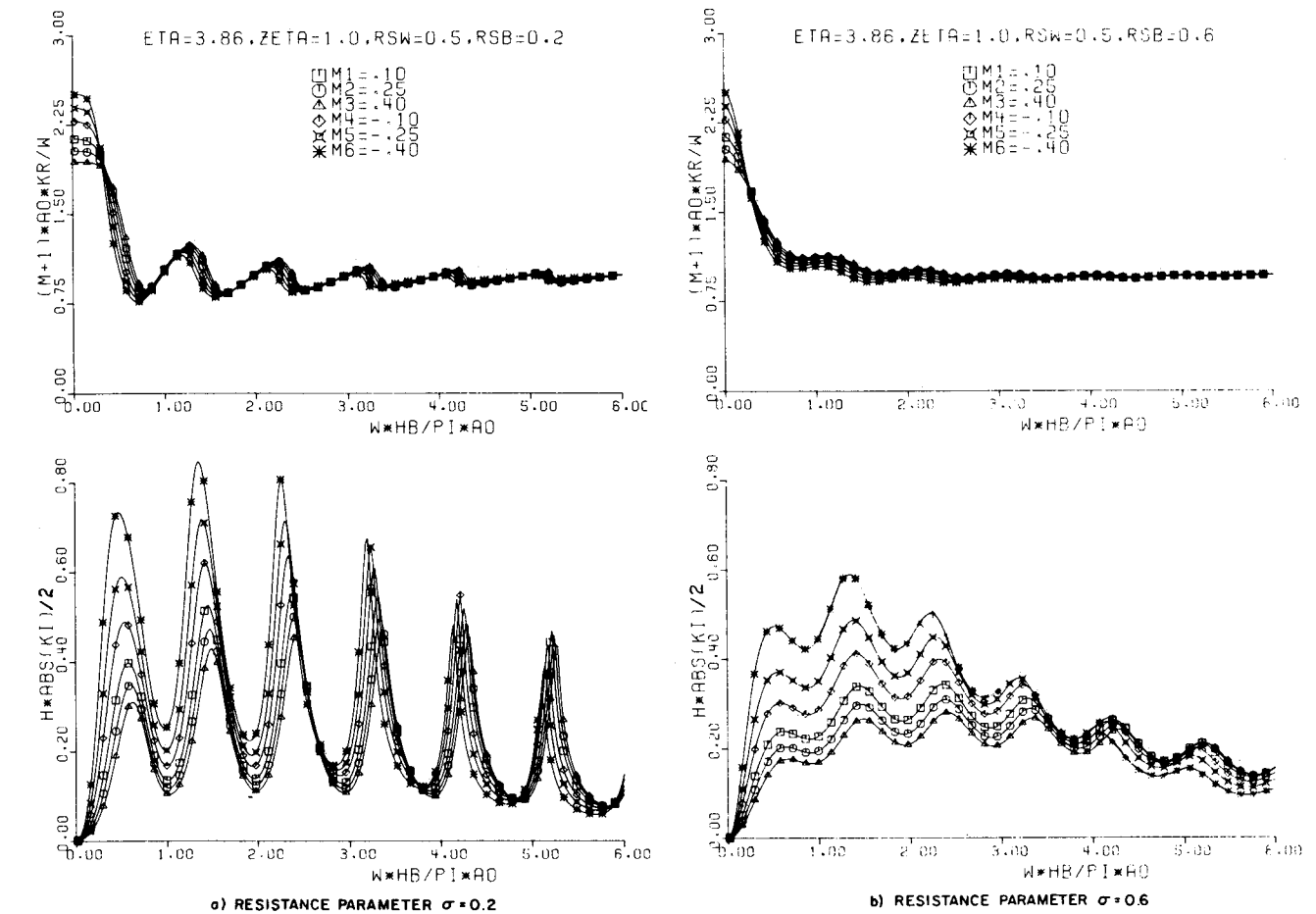
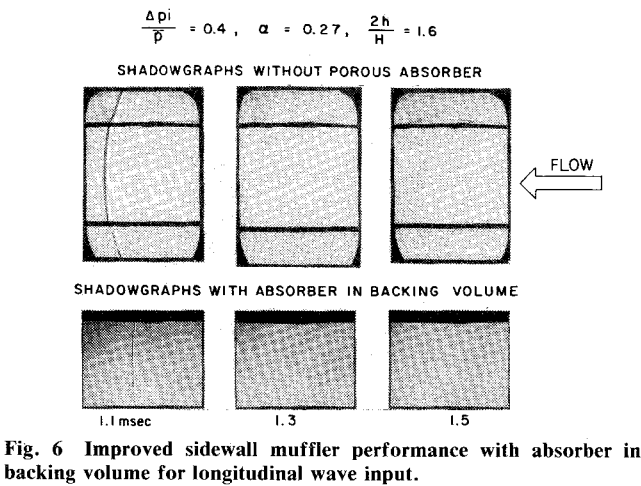
$Q' = 0$ , at least at later times, which could substantially speed restoration of cavity medium homogeneity. Based on theoretical studies, as well as limited experimental work, achieving that ideal appears unattainable with constant flow channel area. It seems that true impedance matching can be achieved by varying the flow channel height as well as the muffler backing depth and open area ratio. To illustrate, when an overpressure pulse enters a muffler section, outflow to the backing volume (at early times) produces a rarefaction wave moving back into the cavity. A compensating compression wave can be produced by simultaneously reducing the flow channel area in the direction of wave propagation. Unfortunately, an appropriate design procedure and experimental

demonstration are not available at this time. It is worth noting that global conservation relations analogous to Eq. (4) can be derived for variable flow channel height.<sup>5</sup>

The last relationship in Eq. (4) shows the expected result (for  $M \rightarrow 0$ ) that the acoustic energy in the incident wave is ultimately either contained in the transmitted and reflected waves or is dissipated within the sidewall muffler, via losses at the perforated sidewall according to one-dimensional theory. Note that much of the dissipation occurs near the cavity since  $|p' - p'_b|^{3/2}$  decreases rapidly as the pulse attenuates. A reasonable objective is to dissipate as much of the acoustic energy as possible. The results given in Fig. 5 show that this is roughly consistent with designing to minimize the transmission coefficient, especially for larger values of  $2h/H$ , but peak dissipation does occur for somewhat smaller values of  $\ell_m/L$ . Since transmission increases rapidly for open area ratios below optimum, the criterion in Eq. (3) is normally preferred. The fraction of energy dissipated within the muffler,  $E_D/E_i$ , can be increased with a longer muffler (i.e.,  $\ell_m/\lambda$ ) largely because greater wavelength stretching is then feasible.

**Other Considerations**

The attenuation process is altered by several effects in a real system. A linear component of sidewall resistance can arise from grazing flow effects and from an acoustic liner over the perforated plate (for efficient boundary layer suction, an interesting option explored in Ref. 5). This leads to reduced shock decay rate at later times which is characterized by the parameter  $\beta = \alpha C_d \zeta_w \sqrt{\gamma p / 2 \Delta p_i}$  where  $\zeta_w$  is a normalized linear sidewall impedance and  $C_d$  is the orifice discharge coefficient. Another effect is fluid inertia within the perforations which causes a delay in achieving the steady-state mass exchange process assumed in the weak wave theory used above.



In consequence, rarefaction waves produced by that process start behind the shock and must catch up by nonlinear wave overtaking. This is characterized in one dimension by the parameter  $K = (\gamma + 1)L\Delta p_i / 8\gamma p d$ , where  $d$  is the hole diameter and  $L$  is the attenuation distance defined in Eq. (1). Further discussion of each of these processes and their modeling can be found in Ref. 5. So long as the system is designed with  $\beta < 0.2$  and  $K > 2$  these effects are not of great importance.

Two-dimensional effects also arise because rarefactions generated at the perforated sidewalls spread as cylindrical wavefronts.<sup>10,13</sup> The transit time from the sidewall to the center of the flow channel will be significant since cavity aspect ratios of one or more are typical. More importantly, the cylindrical wavefronts must overtake the leading shock by nonlinear processes which are weak at the low cavity overpressures characteristic of exciplex lasers. In this case, as well as the examples above, a conservative design requires a somewhat larger attenuator than is indicated by the scaling rules derived from the isolated muffler optimization study. Compressibility effects on the mass exchange process, which are generally minor for exciplex lasers, go in the same direction. These comments are not meant to undermine the muffler scaling rules, as they are quite useful, but to emphasize that they only define starting points for detailed design study.

### III. Primer on Two-Dimensional Acoustics

At the outset the considerations can be viewed as belonging to two categories. First, longitudinal wave attenuation should not be drastically altered by two-dimensional effects over a representative cavity acoustic transit time. In other words, the muffler should display "broadband" performance since a full spectrum of frequencies can be expected. Second, transverse waves refracted out of the cavity by canted sidewalls (cf., Fig. 1) should be rapidly attenuated within the mufflers. Since two-dimensional effects arise via transverse waves, it should not be surprising that similar requirements will result from both considerations.

#### Role of Acoustic Absorber

As seen in the last of Eqs. (4), all dissipation of acoustic energy occurs at the perforated sidewall for a locally reacting muffler according to quasi-one-dimensional theory. This can be a very poor arrangement if it happens that transverse acoustic modes arise which have velocity nodes at or near the perforated sidewalls because that deactivates the dissipation mechanism. The obvious remedy is to put acoustic absorber in the backing volume to obtain dissipation over an extended region. The shadowgraph visualizations during and after shock passage in Fig. 6 illustrate the importance of employing an absorber to achieve good longitudinal wave damping characteristics. Reverberations continue for many acoustic transit times across the flow channel ( $\Delta t > 1$  ms) without absorber, but they are below threshold in 0.2 ms with 3 cgs Rayl/cm absorber in the backing volume. The same shadowgraph sensitivity was used in both experiments.

Acoustic absorber should evidently be placed in the muffler backing volume to improve attenuator performance. The discussion here is restricted to situations in which baffles are placed in the backing volume with spacing small compared to cavity dimensions. Under that circumstance the appropriate absorber resistivity is primarily dictated by transverse wave damping requirements. Otherwise, the absorber must also deal with longitudinal flow induced by the pressure pulse in the backing volume<sup>6</sup> and this additional requirement need not be compatible with the first. A configuration with closely spaced baffles will normally perform better, especially for a close-in muffler arrangement.

Two-dimensional effects on longitudinal waves can be assessed theoretically by considering sinusoidal traveling wave attenuation with absorber in the backing volume of a finite capacity muffler configuration. A solution for the acoustic potential of the form  $e^{i(\omega t - kx)}f(y)$  is sought in the flow chan-

nel, and it consists of a superposition of standing waves between baffles in the backing volume. Here,  $\omega$  is a given radian frequency and  $x$  is in the flow direction. To obtain an eigenvalue problem for  $k$ , linear mass exchange through the perforated sidewall must be assumed. Further discussion can be found in Ref. 5. Results for the real and imaginary parts of  $k$  are given in Fig. 7 for two values of  $\sigma = fH/2\rho a$ , where  $f$  is the absorber resistivity,  $H$  is the flow channel height, and  $\rho a$  is the acoustic impedance of the gas. Also,  $ETA = 2h/H$  is the backing parameter and  $HB = h$  is the backing depth (cf., Fig. 3).

Several conclusions can be drawn from Fig. 7. First, increased resistivity has a moderating influence on both the real and imaginary parts of  $k$ . Note also that the attenuation ( $k_i$ ) has essentially the same mean level for both values of  $\sigma$ , a level that is dictated by the perforated sidewall resistance. The absorber has relatively small effect on longitudinal wave decay (with closely spaced baffles) provided its resistance is sufficiently large. Another interesting feature is the dropoff in damping rate for larger values of input frequency. Careful analysis shows that  $k_i = 0(\omega^{-2})$  as  $\omega \rightarrow 0$ . This appears to be associated with the cylindrical waves mentioned at the end of Sec. II. Namely, it takes a finite time for a wave signaling mass exchange at the perforated sidewall to cross the flow channel.

Low values of  $k_i$  in Fig. 7 tend to arise for integer values of  $\omega h/\pi a = 2h/\lambda$ , where  $\lambda$  is the input wavelength. This corresponds to a transverse velocity node at the perforated sidewall, deactivating that dissipative mechanism. What the absorber basically does is transfer energy out of such poorly damped modes by cross-coupling into components which are effectively attenuated by the perforated sidewall. The mechanism for "broadbanding" is thus subtle. So long as the absorber resistance is high enough, mode-mixing is strong and quasi-one-dimensional performance on longitudinal waves can be approach at frequencies of primary interest.

#### Resistivity Selection

The sinusoidal traveling wave solution discussed above implies no limit to absorber resistivity. However, it does not include an essential feature which is important for the impulsive acoustic input characteristic of exciplex lasers; namely, delayed injection of acoustic energy into the backing volume due to excessive resistance. This implies a tradeoff vs rapid sloshing mode damping, and an optimum absorber resistivity. A definitive procedure for selecting the resistivity is not in hand for a realistic acoustic attenuator configuration, but some useful guidelines can be derived by considering the case of purely transverse waves in a muffler section. That is, the disturbance will be assumed to propagate only in the  $y$  direction, normal to the flow axis.

Modeling of this situation has been given previously,<sup>14</sup> but here the results will be re-interpreted as applying to a symmetric exciplex laser configuration. The essential conclusion for the purpose of this discussion is that an exponential decay envelope can be defined which characterizes restoration of medium optical homogeneity. This provides a convenient means of displaying the dependence of transverse wave decay rate on the ratio of backing depth to flow channel height  $2h/H$ , and on  $\sigma = fH/2\rho a$  where  $f$  is the absorber resistivity. The fit will be stated in terms of an effective reflection coefficient for the perforated sidewalls

$$\left(\frac{\delta\rho}{\rho}\right)_{rms} \leq \frac{\Delta p_i}{\gamma p} \left(\frac{\zeta_w - 1}{\zeta_w + 1}\right)^{at/H} \quad (5)$$

where  $\zeta_w$  is an effective impedance of each sidewall. This form can be derived approximately by imposing Robin boundary conditions,  $p' = \pm \zeta_w \rho a v'$ , on the channel walls; thereby eliminating the backing volume from consideration.

Plots of the effective coefficient obtained in this way are given in Fig. 8. A factor of 10 error in labeling the abscissa was made in the original publication.<sup>14</sup> This has been corrected

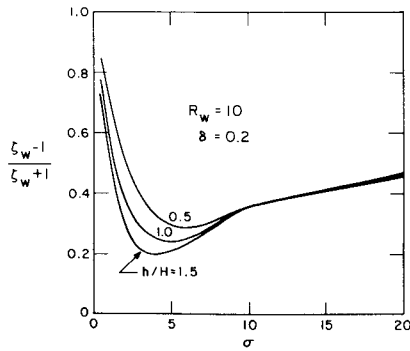


Fig. 8 Effective reflection coefficient of each perforated sidewall.

here. There is seen to be an optimum resistivity for each value of the backing parameter,  $2h/H$ , and the optimum  $\sigma$  decreases as  $2h/H$  increases. An approximate relationship is provided by

$$\sigma_{\text{opt}} \sim 4.5/\sqrt{2h/H} \quad \text{at optimum} \quad (6)$$

where  $\sigma = fH/2\rho a$  assuming porosity and structure factor near one. Although not given in Fig. 8, other calculations done for  $2h/H = 3.8$  have shown that Eq. (6) applies at least over the range  $0.5 \leq 2h/H \leq 4$ . Note from Fig. 8 that  $\sigma \geq \sigma_{\text{opt}}$ , the optimum value, should be chosen since the error of using too low a resistance can be more deleterious. Also, a reflection coefficient of 0.2 or less can be achieved with typical muffler configurations, implying rapid attenuation once transverse waves get into the mufflers.

The theory outlined above presumes linear absorber resistance. In consequence of that and because sidewall resistance has little impact on transverse wave damping, attenuation rate is independent of overpressure amplitude. This will not be the case during early-time longitudinal wave damping, when inertial contributions to absorber resistivity can dominate. However, it usually will be true later in the clearing process when transverse waves are refracted out of the cavity by canted sidewalls therein. The optimum resistance in Eq. (6) is thus for small disturbances. It is also based on an idealized model of the actual transverse wave damping process. Nevertheless, the modeling is useful as a guideline and can be used in other ways as well, as will be seen below.

#### Cavity Transverse Waves

Transverse acoustic waves arise within the laser cavity from nonuniform waste heat deposition, and are a serious concern for the opposed e-beam arrangement required for efficient lasers. Canted sidewalls are generally needed to speed transverse wave clearing, as indicated in Fig. 1. Detailed discussion of canted sidewalls is beyond the scope of this paper. Suffice it to note that restoration of cavity medium homogeneity can be characterized by an exponential decay envelope to good approximation:

$$\frac{\delta p}{p} \sim K \exp\left(-k\Theta_w \frac{at}{\ell_c}\right) \quad (7)$$

where  $k = 2.8/\text{rad}$  according to theory for the first symmetric transverse wave mode<sup>14</sup> and  $k = 3.1/\text{rad}$  according to experiment,<sup>7,15</sup> both for a cavity aspect ratio of one (i.e.,  $\ell_c = H$ ). Here  $\Theta_w$  is the cant angle of each sidewall, not the total included angle as in the theoretical publication.

The main purpose of including Eq. (7) here is to assess the relative significance of muffler contributions to cavity transverse wave decay. Such contributions were demonstrated in the Humdinger, Jr. Flow and Acoustics Facility,<sup>5</sup> which has parallel sidewalls in the cavity, an aspect ratio near one, and mufflers placed very close by. Typical results are shown in Fig. 9 for a muffler with  $2h/H = 3.8$  and an absorber resistance

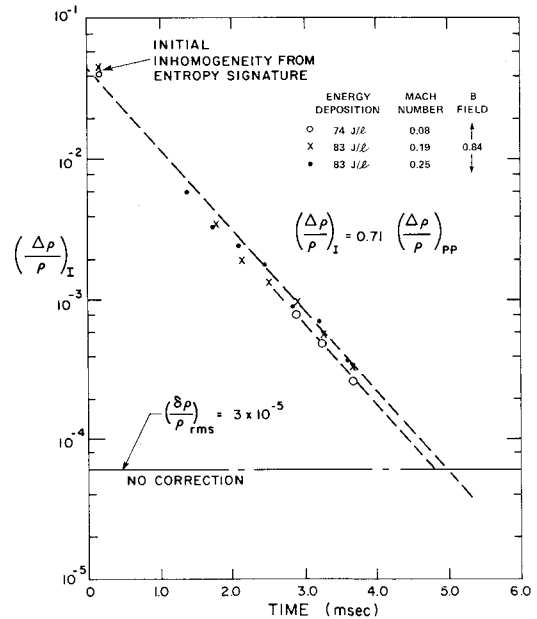


Fig. 9 Attenuation of fundamental transverse channel mode based on movie interferogram reduction.

parameter  $\sigma \sim 0.3$ , and similar results are available in the cited reference for  $\sigma \sim 1.4$ . To adequate approximation, both sets of experimental results are the same, in sharp contrast to the purely transverse wave predictions, and can be correlated as

$$\frac{\delta p}{p} \sim \frac{\Delta p_2}{p} \exp\left(-0.24 \frac{at}{\ell_c}\right) \quad (8)$$

where  $\Delta p_2$  is the initial transverse wave amplitude. Comparing with Eq. (7), this muffler contribution is equivalent to having a 4.5 deg sidewall cant angle and is quite significant.

An approximate theory has been constructed to understand the muffler contribution to damping transverse waves within the cavity. There were two objectives in doing this: 1) explain why altering the absorber resistivity has little effect in the experiments, and 2) establish a basis for predicting performance with altered geometric configuration. The focus is on parallel sidewalls in the cavity as for the Humdinger, Jr. experiments. Under this circumstance, the model must incorporate diffraction to get transverse waves out of the cavity and a damping mechanism within the simulated muffler section. The latter purpose is served by imposing Robin boundary conditions,  $p' = \pm \zeta_w \rho a v'$  with the sign chosen for damping, based on the analogy between a sidewall muffler and a flow channel with "soft" sidewalls established by Eq. (5).

Since the experimental results in Fig. 9 display no discernible dependence on Mach number, it seems adequate to restrict attention to  $M=0$  in constructing a linear acoustics model. Finite Fourier transform analysis is used to conveniently incorporate the inhomogeneous soft wall boundary conditions, and it will be assumed that only one mode  $n \geq 1$  is significant. The last caveat is necessary to render the analysis tractable because all Fourier modes are in fact cross-coupled by the Robin boundary conditions, as is the case for acoustic absorber in the backing volume. It is also true that longitudinal input will excite transverse wave modes via the same mechanism. In short, the discussion which follows is indicative rather than rigorous.

Typical results derived from this model for a symmetric attenuator configuration, an aspect ratio one cavity, and  $n=2$  are shown in Fig. 10, where  $\ell_c$  is the cavity flow length and  $\ell_a$  is the distance between attenuators. A sidewall impedance  $\zeta_w = 1.5$  is chosen based on Fig. 8 and corresponds to a reflection coefficient of 0.2. Opposed e-beam configurations would

entail  $\ell_a \geq \ell_c$ . Cases for  $\ell_a < \ell_c$ , corresponding to an attenuating sidewall segment within the cavity, are included to be sure the model is consistent (excluding diffraction, etc.) with the purely transverse wave analysis for  $\ell_a = 0$ . The medium inhomogeneity  $(\delta\rho/\rho)_{\text{rms}}$ , is obtained by integration over the cavity region without need for correction. It is normalized with respect to the initial level  $\Delta p_2/\gamma p$  where  $\Delta p_2$  is the initial amplitude of the second-order mode, imposed as a top-hat in  $0 \leq x < \ell_c/2$ . Also,  $u_n = 0$  on  $x = 0$  due to symmetry.

The dashed curves in Fig. 10 define an upper bound on cavity medium homogeneity and provide the basis for the exponential decay factors,  $\beta$ , plotted in Fig. 11. There is considerable variation in  $\beta$  with  $\zeta_w$  for small values of  $\ell_a/\ell_c$ , as expected from the purely transverse wave result in Eq. (5). This corresponds to varying absorber resistivity for fixed muffler configuration. However, when the attenuator is wholly outside the cavity, the typical situation, there is very little dependence of  $\beta$  on  $\zeta_w$ . This is true because diffraction of transverse waves out of the cavity becomes the limiting process provided the wall reflection coefficient is not too near one. In the Humdinger, Jr. system  $\ell_a/\ell_c \sim 1$  and the model predicts  $\beta \sim 0.21$  in that case. Comparing with Eq. (8), the model results are seen to be quantitatively, as well as qualitatively, correct for the one condition that can be checked. Note that this theory indicates reduced contribution to transverse wave decay as the mufflers are displaced from the cavity, i.e.,  $\ell_a > \ell_c$ .

#### IV. Entropy Wave Disturbances

There are several sources of entropy waves in a closed cycle laser system. First, waste-heated gas must be swept from the cavity at the flow speed. Of greater concern are late time disturbances arising from pressure wave interaction with flow elements, e.g., screens, honeycomb, heat exchangers, etc. The resulting entropy wave causes density inhomogeneity scaling as

$$\left(\frac{\Delta\rho}{\rho}\right)_{\text{entropy}} = KC_D M \left(\frac{\Delta p}{p}\right)_{\text{acoustic}} \quad (9)$$

where  $K$  is a constant of order 1 dependent on the generation process,  $C_D$  is the drag coefficient of the flow element,  $M$  is the local flow Mach number, and  $\Delta p/p$  is reduced from the in-

itial cavity level by both the muffler and the contraction section. Low Mach number at the dissipative flow elements and small head loss will alleviate muffler requirements. Pursuing the discussion beyond this point is system dependent and beyond the intended scope of the paper.

Entropy disturbances also arise from pressure wave interaction with a sidewall muffler. That will be the main topic of discussion hereafter. The key question is whether entropy production is a bulk process within the flow channel, or whether it is solely associated with gas which has undergone irreversible processes in going into and returning from the muffler backing volume during each pressure cycle of a repetitively pulsed device. Experimental evidence given in the original publication<sup>13</sup> clearly demonstrates that the latter alternative is true. Thus, it is possible to confine the muffler entropy wave to the vicinity of the sidewalls. Adequate confinement offers the possibility of excluding the disturbed region from the optical mode volume to avoid its impact on achievable laser rep-rate.

#### Scaling Arguments

Detailed understanding of the muffler entropy wave is not available at this time, but the following heuristic model should be indicative. Refer to Fig. 3 and consider what happens when a longitudinal pressure pulse in the flow channel passes a particular intra-baffle section. Gas first moves outward into the backing volume when there is overpressure in the flow channel, and it ultimately must all return to the flow channel (without suction) in order to restore the pre-pulse pressure level. In the absence of jet penetration and mixing, this defines an interfacial layer of gas which undergoes dissipation and hence entropy generation through the perforated sidewall. That layer is viewed as the entropy wave in the heuristic model. Convection by the baseline flow is neglected, though that constraint can presumably be removed with suitable integral modeling.

From integral mass conservation,  $\Delta p'_b \sim \alpha \lambda a m/h$  is the pressure rise in the backing volume during the pulse passage time,  $\lambda/a$  with wavelength  $\lambda \sim \ell_c$ . This must be consistent with the mass increase due to inflow and implies an interfacial layer thickness,  $\delta$ . Let  $\Delta p_i$  be the amplitude of the longitudinal wave and assume quasi-steady, incompressible mass exchange through the perforated sidewall. Then the result of the heuristic modeling is

$$\delta \sim \alpha \lambda C_d \sqrt{2\Delta p_i/\gamma p} \quad (10)$$

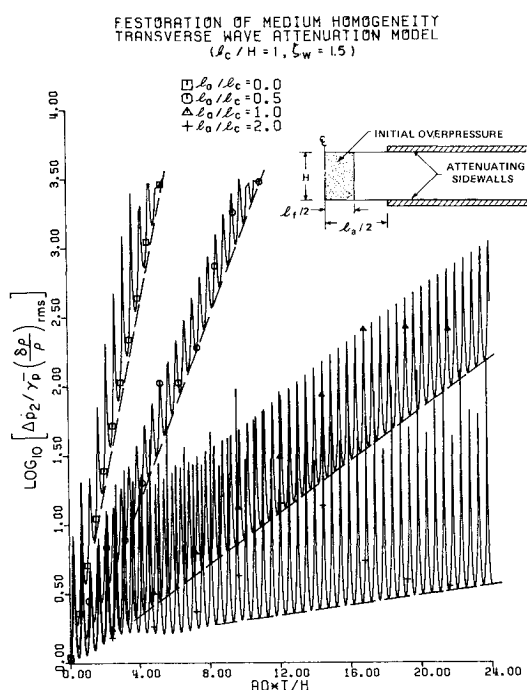


Fig. 10 Decay of second-order transverse wave mode based on model with diffraction and displaced soft sidewalls.

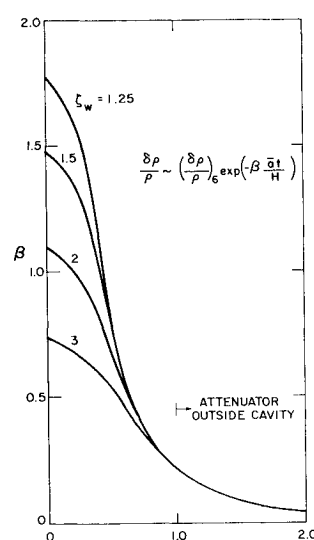


Fig. 11 Variation of exponential decay factor with sidewall impedance and relative location for an aspect ratio one cavity ( $\ell_c = H$ ).



This is in rough accord with the pulsed laser experiments,<sup>5</sup> but appears to underestimate the observed entropy layer thickness. Jet penetration and mixing might account for the difference.

Scaling of the entropy wave with altered conditions can now be assessed. Note first that Eq. (10) estimates the fraction of the flow channel immersed in entropy disturbed gas as

$$\frac{2\delta}{H} \sim \frac{\lambda}{L} \frac{\Delta p_i}{\gamma p} \quad (11)$$

where  $L$  is the characteristic muffler attenuation distance introduced in Eq. (1). Equation (3) is used to eliminate  $L$  in favor of the muffler length,  $\ell_m$ , assuming an optimum configuration. Also  $\lambda \sim \ell_c$ , the cavity length in the flow direction, and the initial cavity overpressure  $\Delta p_c \sim 2\Delta p_i$ . Properly accounting for flow direction, this leads to

$$\frac{2\delta}{H} \sim 1.6(1-M) \frac{\ell_c}{\ell_m} \frac{\Delta p_c}{\gamma p} \quad \text{at optimum} \quad (12)$$

without consideration of jet mixing and penetration effects, things which can presumably be reduced in a large scale device by adjusting the ratio of perforation to cavity dimensions.

As might be expected, decreasing the cavity loading and hence  $\Delta p_c$  should have a favorable effect in reducing the entropy wave thickness. Increasing the muffler length, and thereby decreasing the mass exchange rate for an "optimal" design, is also favorable. Note that the backing parameter for the muffler,  $2h/H$ , has no role according to the model. Representative parameters involve  $\Delta p_c/p \sim 0.3$ ,  $\ell_m/\ell_c \sim 10$ ,  $\gamma = 5/3$ , and  $M \sim 0.2$ , where  $2\delta/H \sim 0.023$  according to Eq. (12). If this scaling relation is correct, it should thus be possible to exclude a region near each sidewall from the optical extraction volume without serious loss in laser system efficiency. This approach will obviously require experimental evaluation.

## V. Conclusion

The physical processes governing acoustic attenuation in a sidewall muffler are numerous and in some cases quite subtle. An attempt has been made here to characterize these and thereby to provide physical understanding fundamental to the design process. By no means is the discussion complete. This is true in part because near-cavity considerations are the focus, avoiding many additional issues that must be considered for a complete closed-loop system. More sophisticated analysis tools are also appropriate in practice, such as a transient two-dimensional flow code capable of handling actual geometries and detailed, nonlinear physical modeling. The material covered primarily serves as a starting point for more detailed studies. The following conclusions can be stated:

1) Leading shock decay distance depends directly on the hydraulic diameter of the flow channel and inversely on the effective open area ratio of the perforated sidewalls of the muffler. The backing depth has no impact on shock decay, to first order, for typical muffler configurations.

2) Quasi-one-dimensional weak wave theory is useful for exciplex lasers because it is a reasonable approximation and involves fewer parameters for a preliminary design study. Once basic muffler characteristics are decided upon, more complete modeling and/or experimental study should be employed for fine tuning.

3) Longitudinal wave attenuation in a finite capacity muffler involves a balance between precursor pulse attenuation and recompression wave strength. This defines an optimum average open area ratio of the perforated sidewalls. Also, for an optimal configuration, acoustic transmission, as well as energy dissipation, depend predominantly on the ratio of cavity volume to muffler backing volume.

4) Global conservation relations show that the total impulse of a pressure wave is preserved through a locally reacting muffler section with constant flow channel height, necessarily implying transmission into the rest of a closed loop flow system. What a well-designed muffler basically does is stretch the in-

put wavelength in such a way that impulse is preserved while dissipating a substantial fraction of the acoustic energy.

5) Acoustic absorber in the muffler backing volume is necessary for good longitudinal wave performance because pulse passage excites transverse sloshing modes. The mechanism underlying this requirement appears to be mode mixing, not direct dissipation. That is, with adequate absorber resistivity to assure strong cross-coupling, energy in sloshing modes which are poorly damped by the perforated sidewalls (due to nearby velocity nodes) is transferred into modes which are well damped. The result is near quasi-one-dimensional behavior for traveling wave frequencies of dominant interest.

6) There is an optimum absorber resistivity for locally reacting mufflers and the pulsed acoustic input of primary concern in exciplex lasers. This entails a balance between rapid damping of sloshing modes vs delayed injection of acoustic energy into the backing volume due to excessive resistance, and the optimum depends on the backing parameter  $2h/H$ .

7) Close-in sidewall mufflers can contribute to transverse acoustic wave attenuation within the cavity for an opposed e-beam configuration. The contribution decreases with muffler displacement from the cavity according to theory and is insensitive to absorber resistivity, both theoretically and experimentally. This is true because diffraction out of the hard-walled cavity becomes a limiting process.

8) Canted cavity sidewalls will typically be required for opposed e-beam configurations to rapidly clear transverse acoustic waves generated by nonuniform waste heat deposition along the e-beam axis.

9) The entropy disturbance arising within the upstream muffler clears the cavity slowly, but it is confined to the vicinity of the sidewalls. This opens the possibility of excluding this entropy wave from the optical mode volume to avoid impact on achievable rep-rate. Scaling arguments indicate an advantage in using longer mufflers to reduce entropy layer thickness.

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